MATH 323 (calc III)

Instructor: Chris E.

Email: eppolito@math.binghamton.edu ★ come and talk.

Ask? Textbook: Multivariable Calculus (don't need a physical copy)

Website: webassign.net. (do need this) — more info later in the email

absolutely need

Gradescope: assignment submission (1st assignment as many times as needed to get a full mark) when ??.

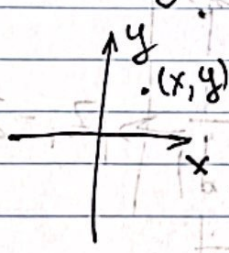
Syllabus: his website (in the email)

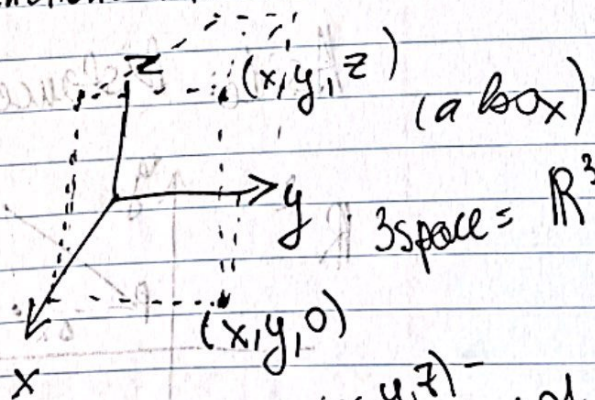
(read) + practice problems

§12.1 Coordinates in 3-Space

IDEA: Extend our calc 1 and 2 to functions with several variables

Some geometry in 3-space





I. coordinates planes

planes where a selected coordinate is 0

(x,y,z) - coordinates of the point $P=(x,y,z)$

a coordinate plane is a set of points in which specified coordinate is 0

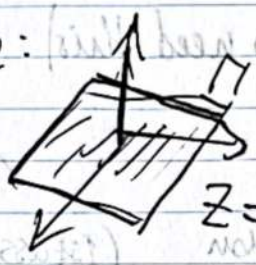
Ex: The xy-plane (aka the $z=0$ plane) in $\mathbb{R}^3$ is

$\Pi = \{P=(x,y,z) \in \mathbb{R}^3 : z=0\}$

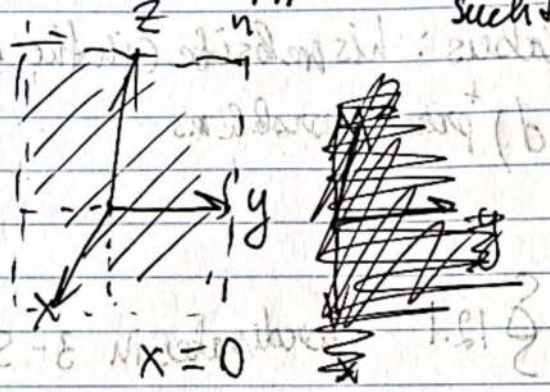
Ex: The yz-plane is

$\{P=(x,y,z) \in \mathbb{R}^3 : x=0\}$

Picture:

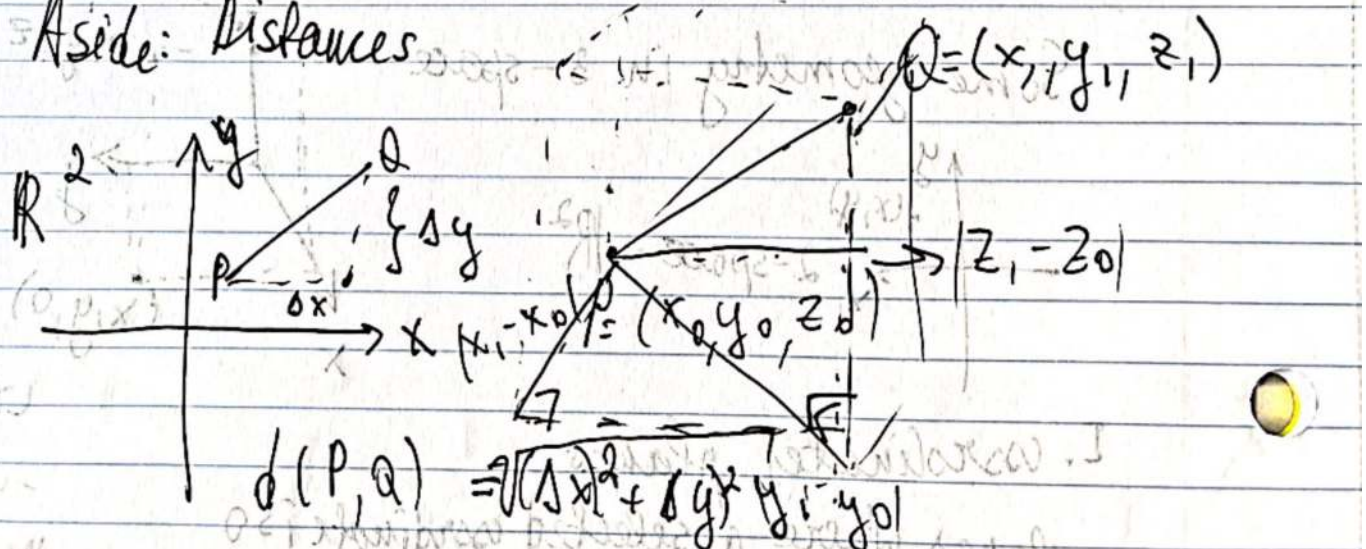


$\in$: "in" ; : "such that"



X → out of the page

Aside: Distances



$\sqrt{x} > 0$ if it exists

in a metric space we do not need to claim absolute values

$$d(P,Q) = \sqrt{(\sqrt{(x_1-x_0)^2+(y_1-y_0)^2})^2 + (z_1-z_0)^2} =$$

$$= \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

Thm (distance formula): For $P(x_0,y_0,z_0)$ and $Q(x_1,y_1,z_1)$ in 3 space the distance between P and Q is

$$d(P,Q) = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

II. Spheres

Let $r>0$ and let $P \in \mathbb{R}^3$

the sphere of radius r centered at P is $S =$

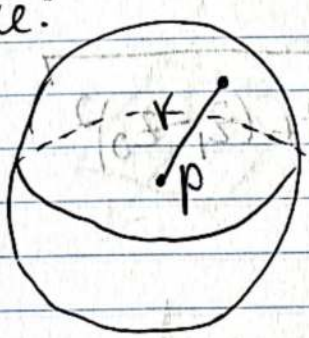
$$= \{Q \in \mathbb{R}^3 : d(P,Q) = r\}$$

If P has coordinates $P = (x_0, y_0, z_0)$, the

$$S = \{Q \in \mathbb{R}^3 : d(P,Q) = r\} = \{(x_1, y_1, z_1) \in \mathbb{R}^3 : \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} = r\} =$$

$$= \{(x_1, y_1, z_1) \in \mathbb{R}^3 : (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 = r^2\}$$

Picture:



NB:

Spheres are "surface of a hollow ball". Not solid

A solid ball is satisfied by

$$(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 \leq r^2$$

NB means "not well"

Note: Everything we have done so far has analogues in higher dimensions as well.

$\{(x_1, y_1, z_1, w) : x_1, y_1, z, w \in \mathbb{R}\}$

$\{(x_1, y_1, z_1, w) : x, y, z, w \in \mathbb{R}\}$

For example (e.g.:), there is a 4-space $\mathbb{R}^4$, and it has a distance formula

$$d((x_0, y_0, z_0, w_0), (x_1, y_1, z_1, w_1)) =$$

$$= \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2+(w_1-w_0)^2}$$

Pictures are hard though

§ 12.2 : Vectors in $\mathbb{R}^3$

Definition: a vector is a directed line segment, where we regard 2 linear segments as equal (or are equivalent) when they are "linear shifts".